INDISCERNIBILITY RELATIONS COMPATIBLE WITH A PARTIALLY ORDERED SET: AN EXAMPLE

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We follow the example introduced in [3]. Consider the following table, reporting a collection of houses for sale in the city of Merate, Lecco, Italy.

House	Price (€)	Size (m^2)	District	Condition	Rooms
a	200.000	50	Centre	excellent	2
b	170.000	70	Centre	poor	3
c	185.000	53	Centre	very good	2
d	190.000	68	Sartirana	very good	3
e	140.000	60	Sartirana	good	2
f	155.000	65	Novate	good	2
g	250.000	85	Novate	excellent	3
h	240.000	75	Novate	excellent	3

In this simple information table eight distinct houses are characterised by five attributes: Price, Size, District, Condition, and Rooms. Let $U = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}, \mathtt{h}\}$ be the set of all houses. We choose the subset of attributes $O = \{\text{Price}, \text{Size}\}$ to define on U a partial order $\leqslant$ as follow. For each $x, y \in U$,

$$x \leqslant y \text{ if and only if } \text{Price}(x) \leqslant \text{Price}(y)\,, \text{Size}(x) \leqslant \text{Size}(y)\,.$$

We obtain the poset $P = (U, \leqslant)$ displayed in Figure 1.

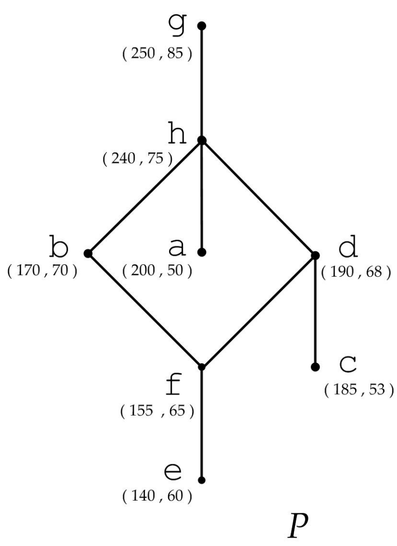

FIGURE 1. $P = (U, \leqslant)$.

We denote by $\mathcal{P}(P, \bar{A})$ the information system having P as universe, and $\bar{A} = \{\text{District}, \text{Condition}, \text{Rooms}\}$ as the set of attributes. Let $D = \{\text{District}\}$, $C = \{\text{Condition}\}$, and $R = \{\text{Rooms}\}$, and denote by π_D, π_C, and π_R the partitions U/I_D, U/I_C, and U/I_R respectively.

Moreover, let $DR = D \cup R$, $CR = C \cup R$, $DC = D \cup C$, $DCR = D \cup C \cup R$ and let $\pi_{DR} = U/I_{DR}$, $\pi_{CR} = U/I_{CR}$, $\pi_{DC} = U/I_{DC}$, $\pi_{DCR} = U/I_{DCR}$. We have:

$\pi_D = \{\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathtt{d}, \mathtt{e}\}, \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}\}$; $\pi_C = \{\{\mathtt{a}, \mathtt{g}, \mathtt{h}\}, \{\mathtt{b}\}, \{\mathtt{c}, \mathtt{d}\}, \{\mathtt{e}, \mathtt{f}\}\}\}$;

$\pi_R = \{\{\mathtt{a}, \mathtt{c}, \mathtt{e}, \mathtt{f}\}, \{\mathtt{b}, \mathtt{d}, \mathtt{g}, \mathtt{h}\}\}$; $\pi_{DR} = \{\{\mathtt{a}, \mathtt{c}\}, \{\mathtt{b}\}, \{\mathtt{d}\}, \{\mathtt{e}\}, \{\mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$;

$\pi_{CR} = \{\{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{c}\}, \{\mathtt{d}\}, \{\mathtt{e}, \mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$; $\pi_{DC} = \pi_{DCR} = \{\{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{c}\}, \{\mathtt{d}\}, \{\mathtt{e}\}, \{\mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$.

Furthermore,

$$\pi_\emptyset = U/I_\emptyset = \{\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}, \mathtt{h}\}\}.$$

Figure 2 represents on P all the partitions listed above.

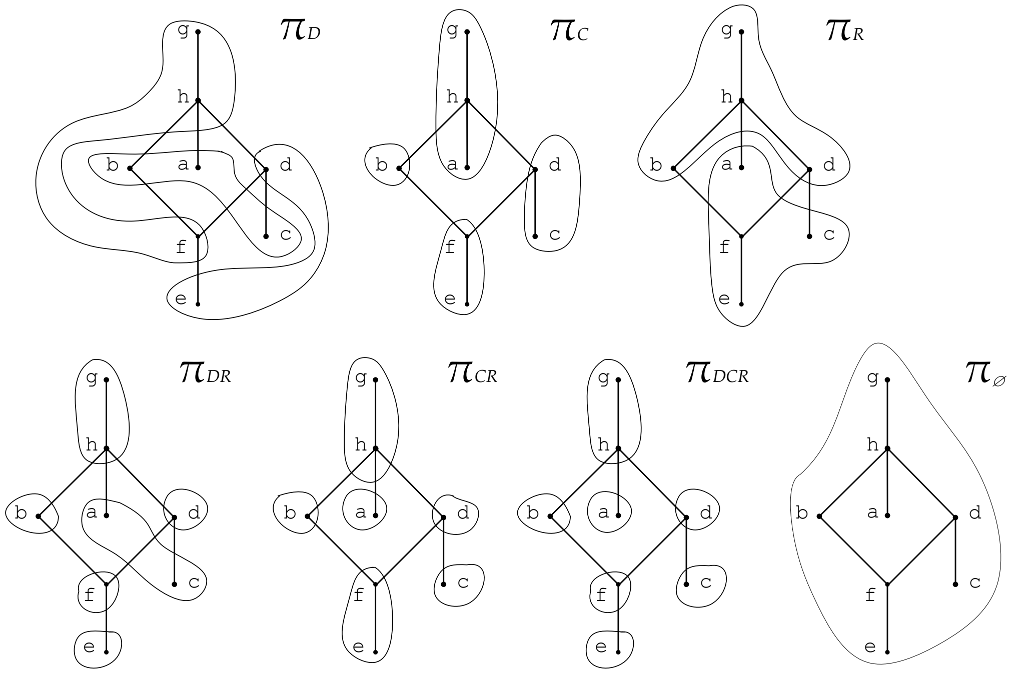

FIGURE 2. Partitions of P induced by indiscernibility relations.

It can be checked that all the partitions, expect π_D, are compatible with P.

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