On the Structure of Indiscernibility Relations Compatible with a Partially Ordered Set

Pietro Codara

Dipartimento di Informatica, Università degli Studi di Milano

Partitions of partially ordered sets

Monotone partition

A monotone partition of a poset $(P, \leqslant)$ is the poset of equivalence classes induced by a preorder $\lesssim$ on P such that $\leqslant \subseteq \lesssim$.

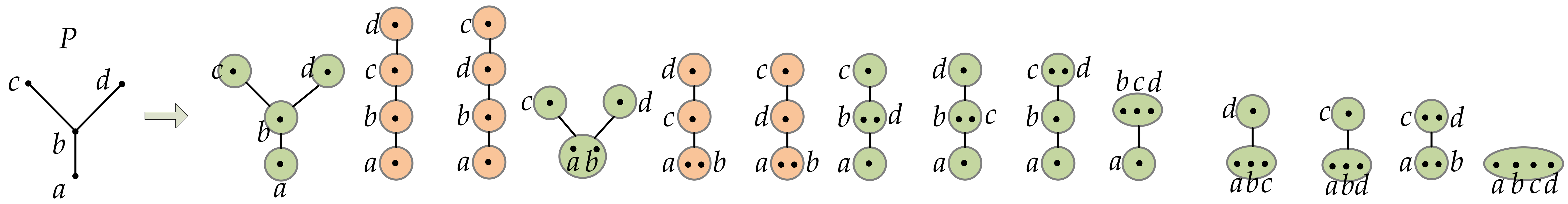
Regular partition

A regular partition of a poset $(P, \leqslant)$ is the poset of equivalence classes induced by a preorder $\lesssim$ on P such that $\leqslant \subseteq \lesssim$, and satisfying

$$\lesssim \; = \; \mathrm{tr}\,(\lesssim \setminus \rho)\,,$$

where $\mathrm{tr}\,(R)$ denotes the transitive closure of the relation R, and ρ is a binary relation defined by

$$\rho = \{(x, y) \in P \times P \mid x \lesssim y,\ x \nleqslant y,\ y \not\lesssim x\}\,.$$



All monotone (regular) partitions of a poset P.

Indiscernibility relations compatible with a partially ordered set

Indiscernibility relations - compatibility

Let $P = (U, \leqslant)$ be a poset, let I_B be an indiscernibility relation on U and let $\pi = U/I_B$. We say I_B is compatible with P if there exists a monotone partition $(\pi, \leqslant)$ of P. Further, if I_B is compatible with P we say that π admits an extension to a monotone partition of P.

Compatibility criterion

Let $(P, \leqslant)$ be a poset and let π be a partition of the set P. For $x, y \in P$, x *is blockwise under* y *with respect to* π, written $x \lesssim_\pi y$, if and only if there exists a sequence $x = x_0, y_0, x_1, y_1, \ldots, x_n, y_n = y \in P$ satisfying the following conditions.

(1) For all $i \in \{0, \ldots, n\}$, $[x_i] = [y_i]$.

(2) For all $i \in \{0, \ldots, n-1\}$, $y_i \leqslant x_{i+1}$.

Let $P = (U, \leqslant)$ be a poset, let I_B be an indiscernibility relation on U and let $\pi = U/I_B$. Then, I_B is compatible with P if and only if, for all $x, y \in P$, $x \lesssim_\pi y$ and $y \lesssim_\pi x$ imply $[x]_\pi = [y]_\pi$.

Let $P = (U, \leqslant)$ be a poset, let I_B be an indiscernibility relation on U and let $\pi = U/I_B$. If π is compatible with P, then π admits a unique extension to a regular partition of P.

11th International Conference on Artificial Intelligence and Soft Computing (ICAISC) - April 30, 2012

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The lattices of partitions of a partially ordered set

Monotone partition lattice

The collection of monotone partitions of $(P, \leqslant)$ is a lattice when partially ordered by set-theoretic inclusion between the corresponding preorders. Specifically, let π_1 and π_2 be the monotone partitions of $(P, \leqslant)$, induced by the preorders $\lesssim_1$ and $\lesssim_2$, respectively. Then $\pi_1 \wedge_m \pi_2$ and $\pi_1 \vee_m \pi_2$ (the lattice meet and join) are the partitions induced, respectively, by the preorders:

$$\lesssim_1 \wedge_m \lesssim_2 = \lesssim_1 \cap \lesssim_2 \,, \quad \lesssim_1 \vee_m \lesssim_2 = \operatorname{tr}(\lesssim_1 \cup \lesssim_2).$$

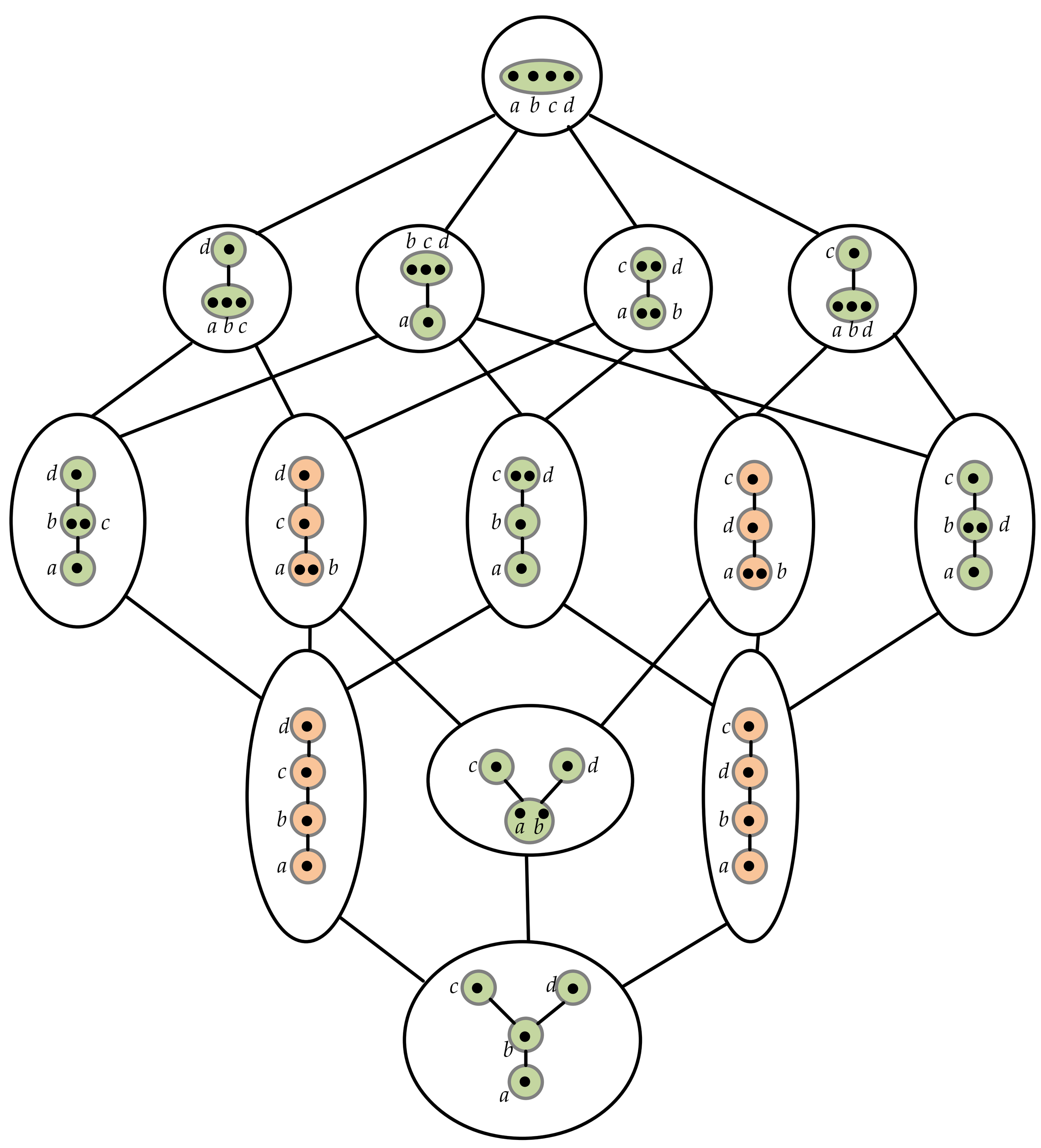
Regular partition lattice

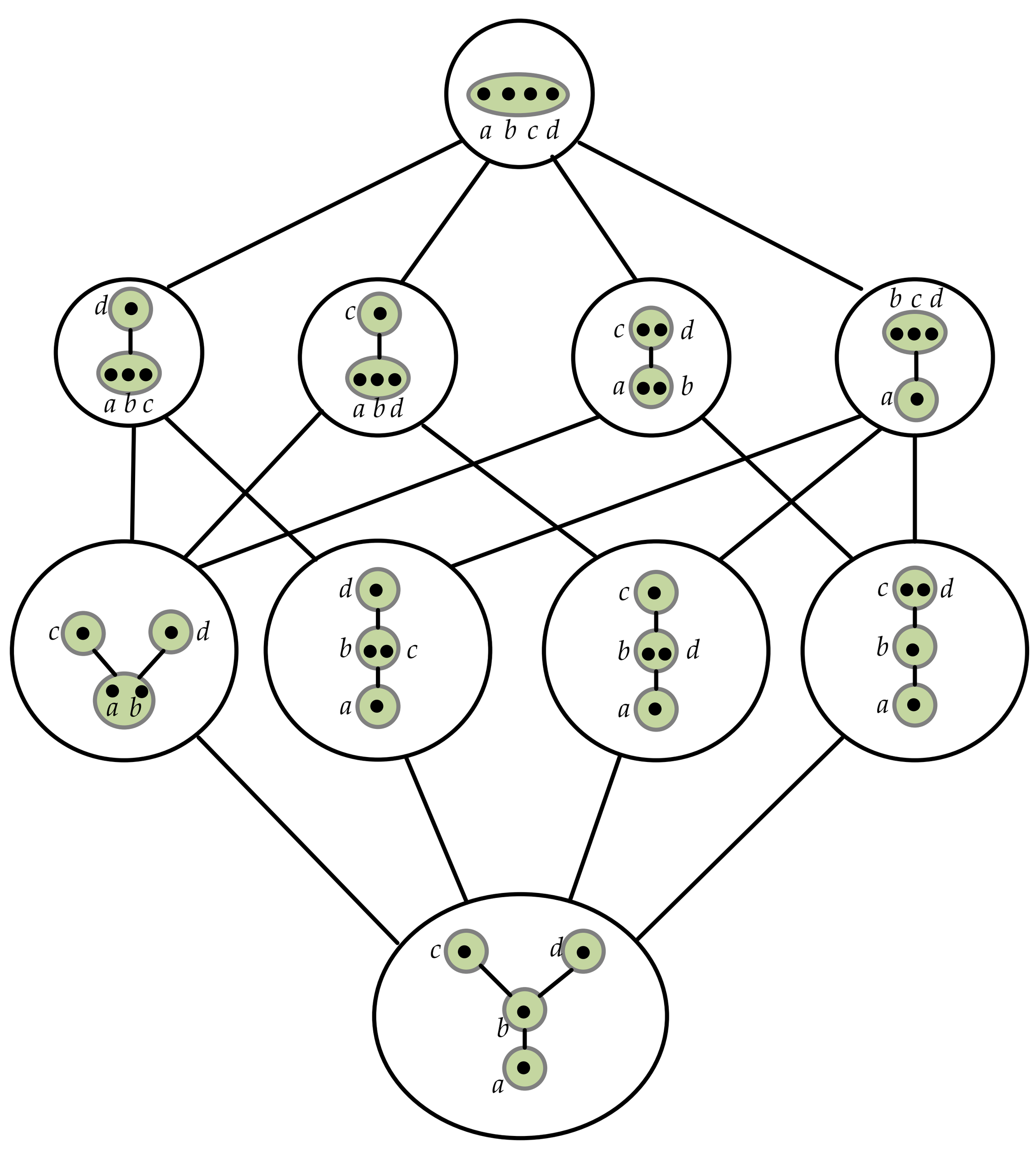
The collection of regular partitions of $(P, \leqslant)$ is a lattice when partially ordered by set-theoretic inclusion between the corresponding quasiorders. Specifically, let π_1 and π_2 be the regular partitions of $(P, \leqslant)$, induced by the preorders $\lesssim_1$ and $\lesssim_2$, respectively, and let

$\tau = \{(x, y) \in (\lesssim_1 \cap \lesssim_2) \setminus \leqslant \ \mid y \not\lesssim_1 x \ \text{ or } \ y \not\lesssim_2 x\}$

Then $\pi_1 \wedge_r \pi_2$ and $\pi_1 \vee_r \pi_2$ (the lattice meet and join) are the partitions induced, respectively, by the preorders:

$$\lesssim_1 \wedge_r \lesssim_2 = \operatorname{tr}((\lesssim_1 \cap \lesssim_2) \setminus \tau)\,, \quad \lesssim_1 \vee_r \lesssim_2 = \operatorname{tr}(\lesssim_1 \cup \lesssim_2).$$





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