On the Structure of Indiscernibility Relations Compatible with a Partially Ordered Set

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Abstract. In a recently published work the author investigates indiscernibility relations on information systems with a partially ordered universe. Specifically, he introduces a notion of compatibility between the (partially ordered) universe and an indiscernibility relation on its support, and establishes a criterion for compatibility. In this paper we make a first step in the direction of investigating the structure of all the indiscernibility relations which satisfy such a compatibility criterion.

Keywords: Indiscernibility Relation; Partially Ordered Set; Partition; Rough Set.

1 Introduction

As stated by Pawlak in [13], the notion of indiscernibility relation stands at the basis of the theory of rough sets. In a recently published paper ([3]), the author investigates the indiscernibility relation in an information system where the universe is partially ordered. After introducing appropriate notions of partition of a partially ordered set (*poset*, for short), a relation between partitions and indiscernibility relations is established. More specifically, the author introduces a notion of compatibility between a poset and an indiscernibility relation on its support, based on the definition of partition of a poset. Further, a criterion for compatibility is established and proved.

In this paper, we make a first step in the direction of investigating the structure of all the indiscernibility relations which are compatible with a poset. To this end, indeed, we need first to learn about the structure of all the partitions of a poset.

Some of the mathematical concepts used in this paper have been developed by the author in [1] and [2]. In these works, the author provides two different notions of partition of a poset, namely, *monotone partition* and *regular partition*. We recall this concepts in Section 2.

Our main results are contained in Section 4, where we describe the lattice structure of monotone and regular partitions of a poset.

In Section 3 we recall the results obtained in [3]. Such results are used in Section 5 to obtain, by way of an example, the structure of all indiscernibility relations compatible with a poset in a specific case.

2 Preorders and Partitions of a Partially Ordered Set

A partition of a set A is a collection of nonempty, pairwise disjoint subsets, often called *blocks*, whose union is A. Equivalently, partitions can be defined by means of *equivalence relations*, by saying that a partition of a set A is the set of equivalence classes of an equivalence relation on A. A third definition of a partition can be given in terms of *fibres* of a surjection: a partition of a set A is the set $\{f^{-1}(y) \mid y \in B\}$ of fibres of a surjection $f : A \to B$.

In [1] and [2], the notion of partition of a poset is investigated. Starting by providing definitions of partition in terms of *fibres* (such kind of definition arise naturally when thinking in terms of categories, *i.e.*, in terms of objects and maps between them), the author provides the corresponding definitions in terms of *blocks*, and in terms of *relations*. In this section, we only present the third kind of definitions, the ones given in terms of relations. Full results and proofs are contained in [1] and [2].

2.1 Two Different Kinds of Partition?

In the case of posets, and in contrast with classical sets, we can derive two different notions of partition. To justify this fact, some remarks on the categories having sets and posets, respectively, as objects, are needed. For background on category theory we refer, *e.g.*, to [9]. Let Set be the category having sets as objects and functions as morphisms. To define a partition of a set in terms of fibres, one makes use of a special class of morphisms of the category Set. In fact, such definition exploits the notion of surjection, which can be shown to coincide in Set with the notion of *epimorphism*. Moreover, in Set, injections coincide with monomorphisms. The well-known fact that each function factorises (in an essentially unique way) as a surjection followed by an injection can be reformulated in categorical terms by saying that the class epi of all epimorphisms and the class mono of all monomorphisms form a factorisation system for Set, or, equivalently, that $(\mathsf{epi},\mathsf{mono})$ is a factorisation system for Set.

Consider the category Pos of posets and *order-preserving maps* (also called *monotone maps*), *i.e.*, functions $f : P \to Q$, with P, Q posets, such that $x \leqslant y$ in P implies $f(x) \leqslant f(y)$ in Q, for each $x, y \in P$. In Pos, $(\mathsf{epi},\mathsf{mono})$ is not a factorisation system; to obtain one we need to isolate a subclass of epimorphisms, called regular epimorphisms. While in Set regular epimorphisms and epimorphisms coincide, that is not the case in Pos. The dual notion of regular epimorphism is *regular monomorphism*. It can be shown (see, *e.g.*, [1, Proposition 2.5]) that $(\mathsf{regular\ epi},\mathsf{mono})$ is a factorisation system for the category Pos. A second factorisation system for Pos is given by the classes of epimorphisms and regular monomorphisms. In other words, each order-preserving map between posets factorises in an essentially unique way both as a regular epimorphism followed by a monomorphism, and as an epimorphism followed by a regular monomorphism.

The existence of two distinct factorisation systems in Pos leads to two different notions of partition of a poset: the first, we call *monotone partition*, is based on the use of epimorphisms, the second, we call *regular partition*, is based on the use of regular epimorphisms.

Remark 1. Another, distinct, notion of partition of a poset can be derived by taking into account the category of posets and *open maps*, instead of the category Pos we are considering. Such kind of partition is called *open partition* (see [2, Definition 4.8]). An application of the notion of open partition can be found in [4].

2.2 Partitions of Partially Ordered Sets

Notation. If π is a partition of a set A, and $a \in A$, we denote by $[a]_\pi$ the block of a in π. When no confusion is possible, we shall write $[a]$ instead of $[a]_\pi$. Further, let us stress our usage of different symbols for representing different types of binary relations. The symbol $\leqslant$ denotes the partial order relation between elements of a poset. A second symbol, $\lesssim$, denotes *preorder relations*, sometimes called *quasiorders*, *i.e*, reflexive and transitive relations.

A preorder relation $\lesssim$ on a set A induces on A an equivalence relation $\equiv$ defined as

$$x \equiv y \text{ if and only if } x \lesssim y \text{ and } y \lesssim x\,, \text{ for any } x, y \in A\,. \tag{1}$$

The set π of equivalence classes of $\equiv$ is a partition of A.

Notation. In the following we denote by $[x]_\lesssim$ the equivalence class (the block) of the element x induced by the preorder $\lesssim$ via the equivalence relation defined in (1).

Further, the preorder $\lesssim$ induces on π a partial order $\preccurlyeq$ defined by

$$x \lesssim y \text{ if and only if } [x]_\lesssim \preccurlyeq [y]_\lesssim\,, \text{ for any } x,\ y \in A\,. \tag{2}$$

We call $(\pi, \preccurlyeq)$ *the poset of equivalence classes induced by* $\lesssim$.

This correspondence allows us to define partitions of a poset (more precisely, monotone and regular partitions) in terms of preorders.

Definition 1 (Monotone partition). *A* monotone partition *of a poset* $(P, \leqslant)$ *is the poset of equivalence classes induced by a preorder* $\lesssim$ *on* P *such that* $\leqslant \subseteq \lesssim$.

Definition 2 (Regular partition). *A* regular partition *of a poset* $(P, \leqslant)$ *is the poset of equivalence classes induced by a preorder* $\lesssim$ *on* P *such that* $\leqslant \subseteq \lesssim$*, and satisfying*

$$\lesssim \;=\; \mathrm{tr}\,(\lesssim \setminus \rho)\,, \tag{3}$$

where $\mathrm{tr}\,(R)$ *denotes the transitive closure of the relation* R*, and* ρ *is a binary relation defined by*

$$\rho = \{(x, y) \in P \times P \mid x \lesssim y,\ x \nleqslant y,\ y \not\lesssim x\}\,. \tag{4}$$

Example 1. We refer to Figure 1, and consider the poset P. One can check, using the characterisations of poset partitions provided in Definitions 1 and 2, that the following hold.

- π_1 is a monotone partition of P, but it is not regular.
- π_2 and π_3 are regular partitions of P, thus monotone ones.

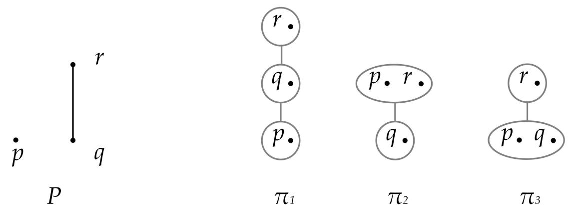

Fig. 1. Example 1.

3 Indiscernibility Relations Compatible with a Partially Ordered Set

Denote by $\mathcal{P} = (P, A)$ the information system having as universe the finite poset $P = (U, \leqslant)$, where U is the collection of objects (the *universe*), $\leqslant$ is a partial order on U, and A is a set of *attributes*.

As in the 'classical' rough set theory, with a subset of attributes $B \subseteq A$ we associate an *indiscernibility relation* on the underlying set of P, denoted by I_B and defined by

$$(x, y) \in I_B \text{ if and only if } a(x) = a(y)\,,$$

for each $a \in A$, and for each $x, y \in P$. Clearly, I_B is an equivalence relation on the underlying set U of P, and thus induces on U a partition $\pi = U/I_B$. We can look at the relation I_B as a way to express the fact that we are unable to observe (to distinguish) individual objects, but we are forced, instead, to think in terms of *granules*, *i.e.*, in terms of blocks of a partition (see, *e.g.*, [10, 14, 15]). In symbols, if $x, y \in P$, x is distinguishable from y if and only if $[x]_\pi \neq [y]_\pi$.

The partition π has no reason to be the underlying set of a partition of P in the sense of Definitions 1 or 2. When it is the case, we say that π *is compatible with the poset*.

In [3] we formalise the notion of compatibility, and proof a criterion for an indiscernibility relation to be compatible with a partially ordered universe. We briefly recall these results.

Definition 3. *Let $P = (U, \leqslant)$ be a poset, let I_B be an indiscernibility relation on U and let $\pi = U/I_B$. We say I_B is* compatible *with P if there exists a monotone partition $(\pi, \preccurlyeq)$ of P. Further, if I_B is compatible with P we say that π admits an* extension *to a monotone partition of P.*

The question arises, under which conditions π can be *extended* to a monotone or regular partition of P, by endowing π with a partial order relation $\preccurlyeq$. In order to give an answer we need a further definition.

Definition 4 (Blockwise preorder). *Let $(P, \leqslant)$ be a poset and let π be a partition of the set P. For $x, y \in P$, x* is blockwise under *y* with respect to *π, written $x \lesssim_\pi y$, if and only if there exists a sequence $x = x_0, y_0, x_1, y_1, \ldots, x_n, y_n = y \in P$ satisfying the following conditions.*

(1) For all $i \in \{0, \ldots, n\}$, $[x_i] = [y_i]$.

(2) For all $i \in \{0, \ldots, n-1\}$, $y_i \leqslant x_{i+1}$.

Corollary 1 (Compatibility Criterion). *Let* $P = (U, \leqslant)$ *be a poset, let* I_B *be an indiscernibility relation on* U *and let* $\pi = U/I_B$. *Then,* I_B *is compatible with* P *if and only if, for all* $x, y \in P$,

$$x \lesssim_\pi y \text{ and } y \lesssim_\pi x \text{ imply } [x]_\pi = [y]_\pi . \tag{5}$$

For regular partitions one can say more: a set partition of P admits at most one extension to a regular partition of the poset P.

Corollary 2. *Let* $P = (U, \leqslant)$ *be a poset, let* I_B *be an indiscernibility relation on* U *and let* $\pi = U/I_B$. *If* π *is compatible with* P*, then* π *admits a unique extension to a regular partition of* P.

The uniqueness property of regular partitions does not hold, in general, for monotone partitions; cf. Figure 2, which shows three distinct monotone partitions of a given poset P having the same underlying set.

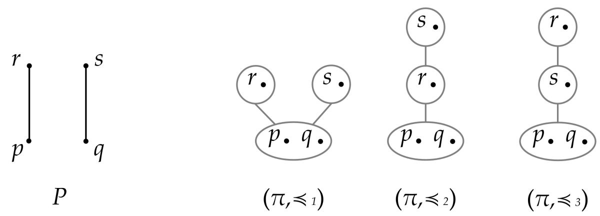

Fig. 2. Distinct monotone partitions with the same support π.

4 The Lattices of Partitions of a Partially Ordered Set

In the light of Definitions 1 and 2, we can think at partitions of a poset as preorders. More precisely, each preorder $\lesssim$ such that $\leqslant \subseteq \lesssim \subseteq P \times P$ defines a unique monotone partition of $(P, \leqslant)$. Moreover, when (and only when) $\lesssim$ satisfies Condition (3) in Definition 2, then $\lesssim$ defines a regular partition of $(P, \leqslant)$. We can endow the set of all monotone (regular) partitions of a poset with a partial order by considering the set-theoretic inclusion between the associated preorders.

Proposition 1. *The collection of monotone partitions of* $(P, \leqslant)$ *is a lattice when partially ordered by set-theoretic inclusion between the corresponding preorders.*

Specifically, let π_1 and π_2 be the monotone partitions of $(P, \leqslant)$, induced by the preorders $\lesssim_1$ and $\lesssim_2$, respectively. Then $\pi_1 \wedge_m \pi_2$ and $\pi_1 \vee_m \pi_2$ (the lattice meet and join) are the partitions induced, respectively, by the preorders:

$$\lesssim_1 \wedge_m \lesssim_2 = \lesssim_1 \cap \lesssim_2\,, \quad \lesssim_1 \vee_m \lesssim_2 = \operatorname{tr}(\lesssim_1 \cup \lesssim_2).$$

Proof. We observe that if $\leqslant \subseteq \lesssim_1$ and $\leqslant \subseteq \lesssim_2$, then $\leqslant \subseteq \lesssim_1 \cap \lesssim_2$, and $\leqslant \subseteq \lesssim_1 \cup \lesssim_2$. We also notice that $\lesssim_1 \cap \lesssim_2 = \lesssim_1$ if and only if $\lesssim_1 \subseteq \lesssim_2$. Moreover, both $\wedge_m$ and $\vee_m$ are idempotent, commutative, and associative, because intersection and union are. Finally, the absorption laws

$$\lesssim_1 \wedge_m (\lesssim_1 \vee_m \lesssim_2) = \lesssim_1 \quad \text{and} \quad \lesssim_1 \vee_m (\lesssim_1 \wedge_m \lesssim_2) = \lesssim_1$$

trivially hold. □

The class of regular partitions also carries a lattice structure.

Proposition 2. *The collection of regular partitions of $(P, \leqslant)$ is a lattice when partially ordered by set-theoretic inclusion between the corresponding quasiorders.*

Specifically, let π_1 and π_2 be the regular partitions of $(P, \leqslant)$, induced by the preorders $\lesssim_1$ and $\lesssim_2$, respectively, and let $\tau = \{(x,y) \in (\lesssim_1 \cap \lesssim_2) \setminus \leqslant \mid y \not\lesssim_1 x \text{ or } y \not\lesssim_2 x\}$ Then $\pi_1 \wedge_r \pi_2$ and $\pi_1 \vee_r \pi_2$ (the lattice meet and join) are the partitions induced, respectively, by the preorders:

$$\lesssim_1 \wedge_r \lesssim_2 = \operatorname{tr}((\lesssim_1 \cap \lesssim_2) \setminus \tau)\,, \quad \lesssim_1 \vee_r \lesssim_2 = \operatorname{tr}(\lesssim_1 \cup \lesssim_2).$$

Proof. By construction, $\lesssim_1 \wedge_r \lesssim_2$ induces a regular partition. We now prove that the preorder $\lesssim_1 \vee_r \lesssim_2$ induces a regular partition, too. Consider $\lesssim_{12} = \lesssim_1 \cup \lesssim_2$, and let $\tau_{12} = \{(x,y) \in \lesssim_{12} \mid x \not\leqslant y,\ y \not\lesssim_{12} x\}$. Suppose $(p,q) \in \tau_{12}$. Say, without loss of generality, $p \lesssim_1 q$. Then, by Definition 2, there exists a sequence $p = z_0 \lesssim_1 z_1 \lesssim_1 \cdots \lesssim_1 z_r = q$ of elements of P such that $(z_i, z_{i+1}) \in \lesssim_1 \setminus \tau_1$ for all $i = 0, \ldots, r$, and $\tau_1 = \{(x,y) \in \lesssim_1 \mid x \not\leqslant y,\ y \not\lesssim_1 x\}$. But if $(z_i, z_{i+1}) \notin \tau_1$, then $z_i \leqslant z_{i+1}$, or $z_{i+1} \lesssim_1 z_i$. In both cases $(z_i, z_{i+1}) \notin \tau_{12}$, and thus $(z_i, z_{i+1}) \in \lesssim_{12} \setminus \tau_{12}$ for all i, and $(p,q) \in \operatorname{tr}((\lesssim_1 \cup \lesssim_2) \setminus \tau_{12})$. Hence, $\operatorname{tr}(\lesssim_1 \cup \lesssim_2)$ corresponds to a regular partition.

We can easily check, by the properties of intersection and union, that $\wedge_r$ and $\vee_r$ are idempotent, commutative, associative, and satisfy the absorption laws. It remains to show that $\lesssim_1 \wedge_r \lesssim_2 = \lesssim_1$ if and only if $\lesssim_1 \subseteq \lesssim_2$. Suppose $\lesssim_1 \subseteq \lesssim_2$. Then, $\lesssim_1 \cap \lesssim_2 = \lesssim_1$ and, since $\lesssim_1$ is regular, $\operatorname{tr}((\lesssim_1 \cap \lesssim_2) \setminus \tau) = \operatorname{tr}(\lesssim_1 \setminus \tau) = \lesssim_1$. Suppose now that $\operatorname{tr}((\lesssim_1 \cap \lesssim_2) \setminus \tau) = \lesssim_1$ and let $x \lesssim_1 y$. Then either $(x,y) \in (\lesssim_1 \cap \lesssim_2) \setminus \tau$, or (x,y) is a pair arising from the transitive closure of $(\lesssim_1 \cap \lesssim_2) \setminus \tau$. In any case, since $(\lesssim_1 \cap \lesssim_2) \setminus \tau \subseteq \lesssim_2$ and $\lesssim_2$ is transitive, we have that $x \lesssim_2 y$, proving that if $\operatorname{tr}((\lesssim_1 \cap \lesssim_2) \setminus \tau) = \lesssim_1$, then $\lesssim_1 \subseteq \lesssim_2$. □

We will call *monotone partition lattice* and *regular partition lattice* the lattices of monotone and regular partitions of a poset, respectively.

5 Structure of Indiscernibility Relations Compatible with a Partially Ordered Set: an Example

Following the example introduced in [3], we show how to obtain the structure of indiscernibility relations compatible with a partially ordered set. Consider the following table, reporting a collection of houses for sale in the city of Merate, Lecco, Italy.

House	Price (€)	Size (m^2)	District	Condition	Rooms
a	200.000	50	Centre	excellent	2
b	170.000	70	Centre	poor	3
c	185.000	53	Centre	very good	2
d	190.000	68	Sartirana	very good	3
e	140.000	60	Sartirana	good	2
f	155.000	65	Novate	good	2
g	250.000	85	Novate	excellent	3
h	240.000	75	Novate	excellent	3

In this simple information table eight distinct houses are characterised by five attributes: Price, Size, District, Condition, and Rooms. Let $U = \{a, b, c, d, e, f, g, h\}$ be the set of all houses. We choose the subset of attributes $O = \{\text{Price}, \text{Size}\}$ to define on U a partial order $\leqslant$ as follow. For each $x, y \in U$,

$$x \leqslant y \text{ if and only if } \text{Price}(x) \leqslant \text{Price}(y)\,, \text{Size}(x) \leqslant \text{Size}(y)\,.$$

We obtain the poset $P = (U, \leqslant)$ displayed in Figure 3.

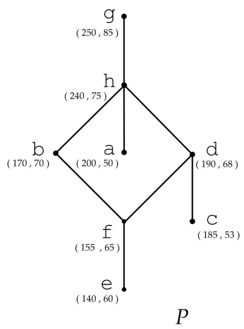

Fig. 3. $P = (U, \leqslant)$.

We denote by $\mathcal{P}(P, \bar{A})$ the information system having P as universe, and $\bar{A} = \{$District, Condition, Rooms$\}$ as the set of attributes. Let $D = \{\text{District}\}$, $C = \{\text{Condition}\}$, and $R = \{\text{Rooms}\}$, and denote by π_D, π_C, and π_R the partitions U/I_D, U/I_C, and U/I_R respectively. Moreover, let $DR = D \cup R$, $CR = C \cup R$, $DC = D \cup C$, $DCR = D \cup C \cup R$ and let $\pi_{DR} = U/I_{DR}$, $\pi_{CR} = U/I_{CR}$, $\pi_{DC} = U/I_{DC}$, $\pi_{DCR} = U/I_{DCR}$. We have:

$\pi_D = \{\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathtt{d}, \mathtt{e}\}, \{\mathtt{f}, \mathtt{g}, \mathtt{h}\}\}$;
$\pi_C = \{\{\mathtt{a}, \mathtt{g}, \mathtt{h}\}, \{\mathtt{b}\}, \{\mathtt{c}, \mathtt{d}\}, \{\mathtt{e}, \mathtt{f}\}\}\}$;
$\pi_R = \{\{\mathtt{a}, \mathtt{c}, \mathtt{e}, \mathtt{f}\}, \{\mathtt{b}, \mathtt{d}, \mathtt{g}, \mathtt{h}\}\}$;
$\pi_{DR} = \{\{\mathtt{a}, \mathtt{c}\}, \{\mathtt{b}\}, \{\mathtt{d}\}, \{\mathtt{e}\}, \{\mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$;
$\pi_{CR} = \{\{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{c}\}, \{\mathtt{d}\}, \{\mathtt{e}, \mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$;
$\pi_{DC} = \pi_{DCR} = \{\{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{c}\}, \{\mathtt{d}\}, \{\mathtt{e}\}, \{\mathtt{f}\}, \{\mathtt{g}, \mathtt{h}\}\}$.

Furthermore,

$\pi_\emptyset = U/I_\emptyset = \{\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}, \mathtt{h}\}\}$.

Figure 4 represents on *P* all the partitions listed above.

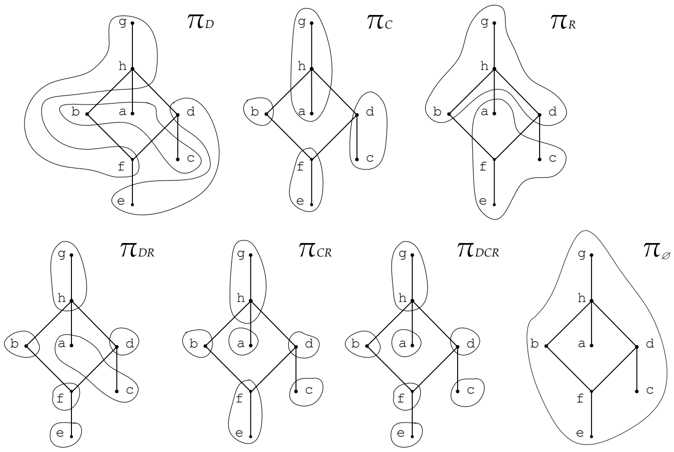

Fig. 4. Partitions of *P* induced by indiscernibility relations.

It can be checked, using Corollary 1, that all the partitions, expect π_D, are compatible with *P*. Figure 5 shows the structure of the indiscernibility relations which are compatible with *P*.

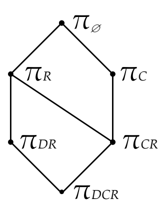

Fig. 5. Structure of the indiscernibility relations compatible with *P*.

6 Conclusion

Rough sets were introduced in the early 1980s by Pawlak ([12]). Since then, lot of works have been published, developing and enriching the theory of rough sets (see, *e.g.*, [8, 11, 14, 16]), and showing how the notion of rough set is suited to solve several problems in different fields of application (see, *e.g.*, [5–7]). The notion of indiscernibility relation stand at the basis of the theory of rough sets.

The results presented in this paper are preparatory to a deeper understanding of the structure of the indiscernibility relations on a partially ordered universe P which satisfies the compatibility criterion introduced in [3]. The main results are Propositions 1 and 2, where we prove that monotone and regular partitions of a poset carry a lattice structure. By way of an example, in Section 5 we show the structure of indiscernibility relations compatible with a specific poset.

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