A Logical Analysis of Mamdani-type Fuzzy Inference, II. An Experiment on the Technical Analysis of Financial Markets

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***Abstract*— This paper is divided into two parts. In Part I, our main objective was to analyse Mamdani-type fuzzy control systems in logical terms, with special emphasis on the fuzzy inference process. To that end, we provided our own inference procedure, cast in the language of standard many-valued logics. We gave an ample discussion of the logical meaning of our procedure. We eventually showed how to fully recover Mamdani-type fuzzy inference from the latter. In this sense, then, our proposal in Part I may be regarded as a logical interpretation of Mamdani-type fuzzy inference. In the present Part II of this paper, we report on the results of an experiment on the technical analysis of the financial markets based on fuzzy techniques. The core algorithm implements the inference procedure described in the first part of the paper. The experimental results support the claim that our theoretical analysis in Part I provides a sound interpretation of Mamdani-type fuzzy inference.**

I. Introduction

This is the latter half of a two-part paper. The first part [1] provides a theoretical analysis of Mamdani-type fuzzy control systems in logical terms, with special emphasis on the fuzzy inference process. In the present part, empirical in character, we report on the results of an experiment on the technical analysis of the financial markets based on fuzzy techniques. The core algorithm implements the inference procedure described in the first part of the paper. By the end of the paper, we will argue that the experimental results support the claim that our theoretical analysis in Part I provides a sound interpretation of Mamdani-type fuzzy inference.

II. Background on Technical Analysis

In the context of trading the financial markets, the term *technical analysis* refers to a set of techniques based on the key hypothesis that reliable forecasts about the markets' future trend can actually be obtained through the analysis of the market's past behaviour alone. The literature on the subject is vast; here we only provide a small sample of references. [2] adopts a statistical approach to the analysis of information encoded into prices' historical data and the influence they exert on investors' behaviour; [3] presents a formalisation of some technical analysis concepts along with a comparison against the Wiener-Kolmogorov model; [4] compares strategies based on indicators with four renowned statistical models; finally, [5] is an attempt at combining technical analysis and prediction models based on time series.

A. Financial Trading

The basic operation we are interested in is a *trade*. In the experiment we report upon here, we trade ten different markets, listed in Table II. They vary in typology, behaviour, exchange, and base currency. A trade may be thought of as a single transaction made up of an open and a close move. If the trade's effect is to purchase, it is said to be *long*; if its effect is to sell, it is said to be *short*. We always assume that all our trades allow both long and short positions to be taken. For a fixed trade amount, the quantity traded (say, the number of contracts) depends on the price of the market at opening and closing time, and determines the investor's exposure. Thus, since each contract has a fixed monetary value, we can compute each trade's contribution to the increment or decrement of the initial investment, according to the market's movement. More precisely, at each instant of time i we compute the *equity run* to gauge the current value of our capital, namely

$$er_i = er_{i-1} + opl_i,$$

where opl_i, which stands for *open profit and loss*, is the value of the current net exposure (i.e. of all the open trades) and $er_0 = C$, with C the starting capital. In our experiment we use a starting capital of 10 million units and a per-trade investment of 1 million units, where units are market dependent. An example of equity run is provided in Figure 1 for four different trading strategies to be discussed later.

From the equity run we can obtain further information, namely, the entity of the loss or gain for a specified period. This is simply called *return* and is defined as the variation in the equity run's value between two instants in time, that is

$$r = (er_f - er_i)/er_i.$$

On the other hand, the *volatility*, which in this paper will be a synonym for standard deviation, provides instead information about the strategy's returns' variability.

Finally, it is desirable to have some way to measure losses and their duration. This is achieved by the *drawdown*, i.e. the percentage loss relative to the current value of the equity run with respect to its last maximum. It provides two significant pieces of information:

1) the *worst drawdown*, i.e. the worst loss attained, and
2) the *recovery time*, i.e. the longest losing period.

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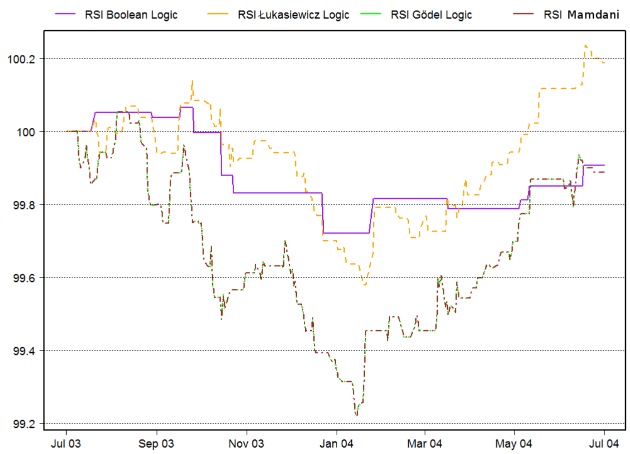

Fig. 1. Equity runs computed on the Euro/Dollar market, using various versions of the RSI indicator, July 2003 to July 2004. Numbers are percentages.

Figure 2 shows the drawdowns for the equity runs in Figure 1.

B. Indicators

Indicators are tools used by analysts in order to forecast the markets' trend. Relevant to this paper are the following two.

1) MACD: Moving Average Convergence Divergence. Introduced by Gerald Appel in the 1970s (see [6]), this indicator tries to forecast market trends by comparing short and long-term tendencies. These are modelled by two exponential moving averages whose standard parameters are 12 and 26 periods, respectively. The long-term average is subtracted from the short-term average, and the result is called MACD. The *signal line* is computed out of MACD: it is again an exponential moving average, usually of 9 periods. Trading signals are produced according to the schema provided below.

- When the MACD crosses the signal line from below, buy.
- When the MACD crosses the signal line from above, sell.

The rationale behind this is based on the observation that a falling MACD shows a weakening of an uprising trend, in that the short-term trend is weakening faster than the long-term one. It is expedient to further compute the difference between MACD and the signal line, obtaining what is known as the *convergence divergence oscillator*, or *cdo* for short. This swings around the zero value, with positive (negative) values meaning the MACD is above (below) the signal line. Thus, the schema above reads as follows in terms of the *cdo*. *When cdo turns positive, buy; when cdo turns negative, sell.*

2) RSI: Relative Strength Index. Developed by James W. Wilder in [7], this indicator attempts to forecast the market's behaviour by looking at the extent of recent gains and losses to identify situations described as *overbought* and *oversold*.

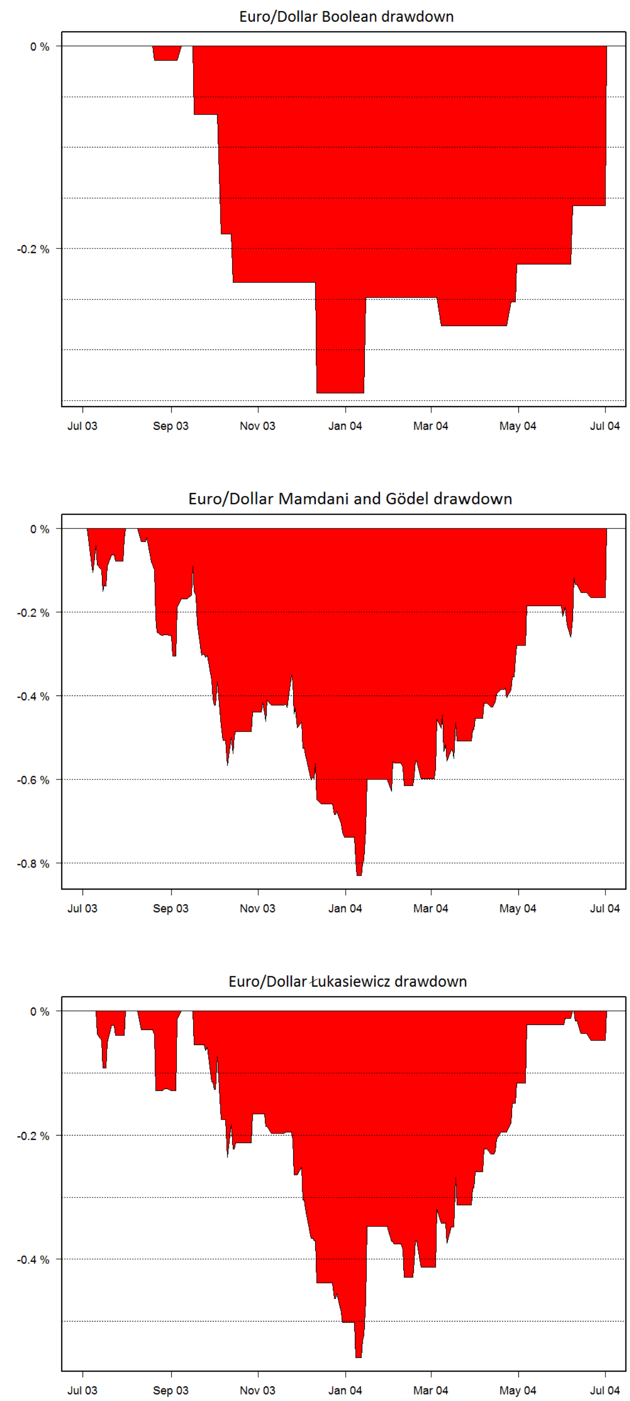

Fig. 2. Drawdown calculated on the Euro/Dollar market, July 2003 to July 2004. Note how Mamdani and Gödel coincide, because the respective equity runs coincide.

It is defined as

$$\mathrm{RSI} = 100 - \frac{100}{1 + \mathrm{RS}},$$

where RS (relative strength) is the ratio between the averages of upward and downward closing prices of the last n periods. A closing price is said to be upward if it is greater than the previous day's price, and downward when it is smaller. Usually $n = 14$. Two conventional thresholds act as flagpoles for oversold and overbought situations. Standard values for the thresholds are 30 and 70. Signals are generated when the following conditions occur (RSI_i is the value of RSI at instant i):

- if $\mathrm{RSI}_i > 30$ and $\mathrm{RSI}_{i-1} \leq 30$, buy;
- if $\mathrm{RSI}_i < 70$ and $\mathrm{RSI}_{i-1} \geq 70$, sell.

The general idea behind the first rule is that values near the lower threshold signal an underpriced market, likely to reverse its trend; and similarly for the second rule. Figure 3 shows an example of these indicators for the Euro/Dollar market.

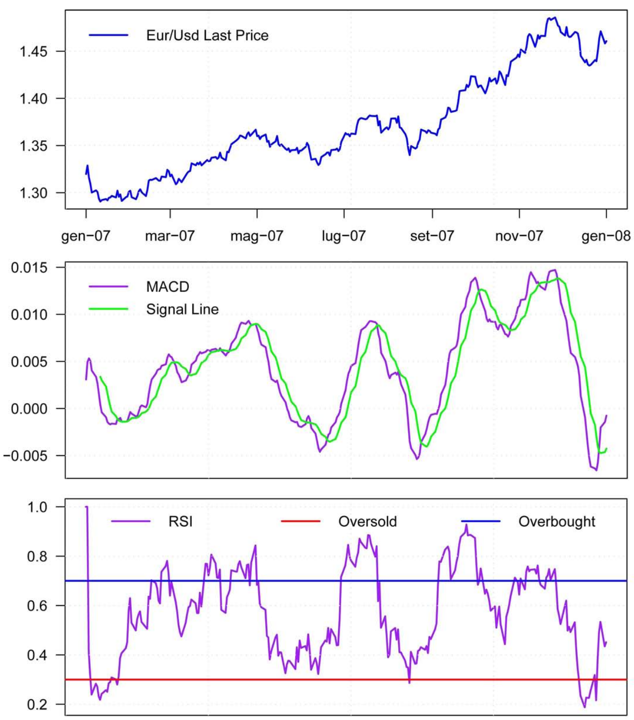

Fig. 3. MACD and RSI indicators computed on the Euro/Dollar market, year 2007.

III. System Description

The indicators discussed above work in a Boolean fashion – at specific instants of time, they generate signals to the effect that the trader either goes long, or goes short all the way. On the fuzzy approach, by contrast, one would adjust market exposure in a continuous manner. To achieve this, we assume indicators do provide signals at each instant of time, although the *strength* of signals may be varying. Classical signals will then correspond to strong fuzzy signals, whereas their absence will generally correspond to a weak (possibly nil) fuzzy signal. Market exposure will be adjusted accordingly, which requires countenance of intermediate possibilities between totally long and totally short positions. To make this precise, we need to *fuzzify* both indicators and position. This will require the normalisation of the respective ranges of the latter quantities, as detailed below.[1] By convention, we always normalise to the real unit interval $[0, 1]$. Below, we introduce a Mamdani-type fuzzy control system for trading with RSI and MACD indicators. We then introduced an alternative system based on the contents of Part I of this paper.

A. Fuzzification

1) Fuzzy MACD: As a preliminary to the fuzzification of the MACD indicator, we need to normalise its values to $[0, 1]$. First, given a time series, we compute the maximum divergence (in absolute value) cdo_{max} observed over a number of past periods. We then assume as the range to be normalised the interval $[-cdo_{max}, cdo_{max}]$. In computing cdo_{max}, however, we do not want to look too far back into the past of the time series, for it can be argued that this would make signals unreliable. To see this, recall that the MACD indicator is parametrised by the numbers of the various past periods over which averages are computed. In the present setting, the largest span considered is twenty-six periods long, and therefore the current value of the indicator only depends on the past twenty-six data items. It follows that using older data items in the normalisation process introduces spurious information. Summing up, we normalise the value of cdo_i – the cdo at time i – by

$$ncdo_i = \frac{1}{2}\left(1 + \frac{cdo_i}{cdo_{max}}\right).$$

Note that when $cdo_i = 0$, the outcome of the normalisation is 0.5.

We can now turn to the fuzzification proper. We only use two fuzzy sets, called NEG ("negative") and POS ("positive"). They are shown in Fig. 4. This specific choice of NEG and POS aims at smoothing the transition from a fully negative cdo to a fully positive cdo passing through a nil cdo. In the crisp setting the actual magnitude of cdo is immaterial – only its sign matters. Here, the choice of NEG and POS reflects the assumption that the magnitude of cdo does carry information whenever it is sufficiently close to zero. For instance, an increase of magnitude from near below zero to a value even closer to zero, even if not a positive one, may indicate a tendency to eventually cross the zero boundary, though the strength of that indication is weaker than an actual sign inversion. In terms of the normalised indicator $ncdo$ – the abscissa in Figure 4 – one needs to recast the former remarks taking into account that the zero line is here placed at 0.5. Accordingly, the sentences '*The* $ncdo$ *is positive*' and '*The* $ncdo$ *is negative*' are allowed to take on

[1]While choosing an appropriate, non-distortive normalisation can notoriously prove a challenging task in practice, in this paper we are not concerned with this issue at a general level.

intermediate truth values when $ncdo$ lies in the open interval $(0.45, 0.55)$. The interval is relatively narrow because it is hard to argue that the precise magnitude of the $ncdo$ far away from the zero line actually does carry significant information. A certain amount of tuning was performed to back this up empirically, even though extensive fine-tuning of parameters is obviously not the focus of this piece of work.

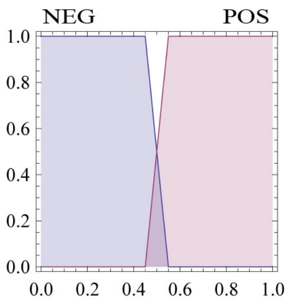

Fig. 4. Fuzzy sets for the MACD indicator (in abscissa the normalised *cdo*.)

2) Fuzzy RSI: The RSI is bounded by definition to $[0, 100]$, and we normalise its values to [0,1] by setting nRSI $=$ RSI$/100$.

In order to proceed with the fuzzification of RSI, observe that this indicator relies on three crisp bands in order to trigger trading signals. Namely, if RSI $\geq$ 70 the market is *overbought*; if RSI $\leq$ 30 the market is *oversold*; and if 30 $<$ RSI $<$ 70 the market is neither overbought nor oversold – we say it is *average*. Our fuzzification aims for smoother bands, by assuming that the sentences "*The market is overbought*", "*The market is oversold*", and "*The market is average*" have intermediate truth values in neighbourhoods of the thresholds 30 and 70. A choice of fuzzy sets consistent with the foregoing is shown in Figure 5. Here, in abscissa one reads the normalised RSI.

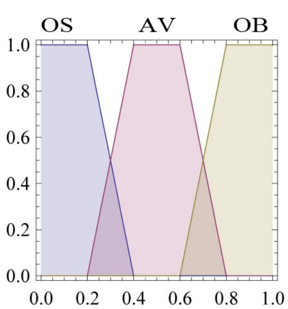

Fig. 5. Fuzzy sets for the RSI indicator (in abscissa the normalised RSI.)

3) Fuzzy Position: Recall from Sect. II-A that the maximum per-trade investment is 10^6 units; this yields a maximal market exposure, say $maxpos$. A classical trading system can take three different positions on the market. Namely, the position pos can be all the way long ($pos = +maxpos$), all the way short ($pos = -maxpos$), or else one can stay out of the market ($pos = 0$). As mentioned before, we want to be able to adjust market exposure in a continuous

Operator	Łukasiewicz	Gödel	Mamdani
conjunction	$\max(0, a+b-1)$	$\min(a, b)$	$\min(a, b)$
disjunction	$\min(1, a+b)$	$\max(a, b)$	$\max(a, b)$
negation	$1-a$	1 if $a=0$, 0 otherwise	$1-a$
if…then…	$\min(1, 1-a+b)$	1 if $a \leq b$, b otherwise	$\min(a, b)$
aggregation	$\max(0, a+b-1)$	$\min(a, b)$	$\max(a, b)$

TABLE I

The definition of operators for the systems used in the simulation.

manner, that is, to take intermediate positions in the interval $[-maxpos, +maxpos]$. As for the indicators, the first step is normalisation. The natural choice here is to normalise the interval $[-maxpos, +maxpos]$ to $[0, 1]$ linearly. Therefore, a normalised position of 0.5 corresponds to the nil position (out of the market). Fuzzy sets for the position are shown in Figure 6, where L is the fuzzy set which gives truth values to the sentence '*Position is Long*', and S is the fuzzy set which gives truth values to the sentence '*Position is Short*'.

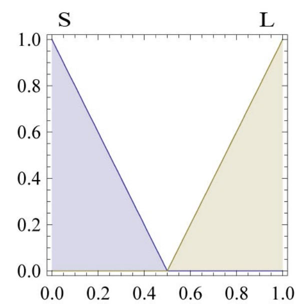

Fig. 6. Fuzzy sets defined for the position (in abscissa the normalised position.)

B. Mamdani-type Fuzzy Inference

We assume familiarity with Mamdani-type fuzzy control systems in this section. For details, please see the first part of this paper [1]. In the experiment we are reporting upon, we used the standard operators shown in Table I. We observe that the 'if…then…' operator is interpreted by the semantics of implication in the case of Łukasiewicz and Gödel logic, whereas it is interpreted by the minimum in Mamdani-type systems. As is well known, the minimum cannot be regarded as the semantics of a genuine logical implication if one works in the setting of residuated structures, where implication is the residuum of conjunction. For details, please see [8].

Let us now formalise the control theories for the MACD and RSI indicators.

As for the MACD, we observe that its behaviour can be captured by the following sentences:

- "when cdo turns from negative to positive, buy" and
- "when cdo turns from positive to negative, sell".

Consequently, two simple rules are all that we need:

IF cdo_{i-1} is NEG AND cdo_i is POS THEN *pos* is L
IF cdo_{i-1} is POS AND cdo_i is NEG THEN *pos* is S

Here NEG and POS stand for negative and positive, while cdo_{i-1} and cdo_i represent the previous and current values of the *cdo*, respectively. L and S stand for long and short.

Concerning the RSI, the indicator's control theory is reported below, with OS, AV, OB standing for oversold, average, and overbought.

IF rsi_{i-1} is OS AND rsi_i is AV THEN *pos* is L
IF rsi_{i-1} is OS AND rsi_i is OB THEN *pos* is L
IF rsi_{i-1} is OB AND rsi_i is AV THEN *pos* is S
IF rsi_{i-1} is OB AND rsi_i is OS THEN *pos* is S

The first two rules govern the triggering of long signals. The remaining two govern the triggering of short signals. Sect. IV-A provides some insights into the input/output behaviour of the systems based on the rules above.

C. Logic-based Alternative to the Mamdani-type Inference

The basis of our alternative system is the interpretation of a set of fuzzy rules as a theory, i.e. a set of formulas, in a many-valued logic L. Full details may be found in Part I. For the reader's convenience, we provide a rough summary of the procedure here.

We obtain a theory Θ from a set of fuzzy rules, as follows.

- Each sentence of the form '*x is X*' is interpreted as a logical variable X.
- The connectives AND, OR, NOT, and THEN used in the fuzzy rules are interpreted as the corresponding logical connectives of the logic L, and aggregation of rules is achieved via the conjunction of L.

Having defined Θ, we proceed to assign real values to the *input* variables of our theory (i.e. we define a partial assignment $\bar{\mu}$ to the input variables). This is done with the use of fuzzy sets defined for the input observables (see Sect. III-A). The aim of the control system is to find the 'best' possible value for the output observable. In our approach, this corresponds to extracting one value from the set of all values of the *output* variables which maximise the truth value of the theory Θ. This can be done by:

1) Extending $\bar{\mu}$ to a partial assignment μ which assigns truth values to each output variable, by using the fuzzy sets defined for the output observable;
2) computing the truth function of the theory Θ under the assignment μ;
3) finding a set of complete assignments which maximise the truth value of the theory; and
4) choosing a defuzzification method to extract a single assignment from the set of all maximising assignments.

Let us revert to our experiment. Consider the set of rules for the MACD-based fuzzy control system described in the previous section. We rewrite such rules as a theory Θ_{MACD}, namely,

$$(NEG_{i-1} \wedge POS_i \rightarrow L) \wedge (POS_{i-1} \wedge NEG_i \rightarrow S)\,.$$

As to the RSI-based fuzzy control system, we also rewrite the fuzzy rules defined in the previous section as a theory Θ_{RSI}, namely,

$$(OS_{i-1} \wedge AV_i \rightarrow L) \wedge (OS_{i-1} \wedge OB_i \rightarrow L) \wedge \\ \wedge (OB_{i-1} \wedge AV_i \rightarrow S) \wedge (OB_{i-1} \wedge OS_i \rightarrow S)\,.$$

In the experiment, the logic-based approach for the MACD and RSI control systems was tested by interpreting the respective theories in two well-known many-valued logics: Gödel logic, and Łukasiewicz logic. The semantics of the connectives is provided in Table I.

Algorithm 1 shows how the position is computed in the case of the RSI-based strategy. An analogous algorithm has been used for the MACD-based strategy. The algorithm works as follows. The inputs rsi_{i-1} and rsi_i are used to compute the truth values to be assigned to the variables OS_{i-1}, OB_{i-1}, OS_i, OB_i, AV_i: values are assigned by the function getMembership, using the fuzzy sets for RSI. Next, we compute all possible pairs of truth values for the variables L and S, for values of position ranging in $[0, 1]$ by steps of 0.01. For each pair of values assigned to L and S we evaluate the theory Θ_{RSI}. When the theory achieves a maximal truth value under the assignment, we add the current position (i.e. the position that has generated the assignment) to the vector P of maximising positions. Finally, we extract one of the maximising positions from P, by computing the arithmetic mean of the values of P. (This latter step corresponds to the well-known mean of maxima defuzzification method).

Algorithm 1: The algorithm for the computation of the position, RSI case. Θ_{RSI} is the control theory.

input : rsi_{i-1}, rsi_i
output: *pos* ; // output position
OS_{i-1}, OB_{i-1} $\leftarrow$ getMembership(rsi_{i-1});
OS_i, OB_i, AV_i $\leftarrow$ getMembership(rsi_i);
$\bar{\mu} \leftarrow \{OS_{i-1}, OS_i, OB_{i-1}, OB_i, AV_i\}$;
pos, $v_{max} \leftarrow 0$;
/* initialize vector P of maximising positions */
$P \leftarrow \emptyset$;
while $pos \leq 1$ **do**
  S, L $\leftarrow$ getMembership(pos);
  $\mu \leftarrow \bar{\mu} \cup \{S, L\}$;
  $v \leftarrow$ evaluate(Θ_{RSI}, μ);
  if $v > v_{max}$ **then**
    $v_{max} \leftarrow v$;
    $P \leftarrow \{ pos \}$;
  else if $v = v_{max}$ **then**
    $P \leftarrow P \cup \{ pos \}$;
  end
  $pos \leftarrow pos + 0.01$;
end
return mean(P);

D. Defuzzification

In the Mamdani-type fuzzy control system (see Sect. III-B) the defuzzification process takes as input a fuzzy set, and provide as output a value for pos. The output value can be computed in a number of ways. MATLAB's Fuzzy Logic Toolbox provides five different standard defuzzification methods: centroid, bisector, mom (mean of maxima), lom (largest of maxima), and som (smallest of maxima). We work with the mean of maxima defuzzification method. As detailed in Part I of this paper [1], such a choice affords an easier comparison between the Mamdani-type fuzzy system and the system based on our approach form the standpoint of fuzzy inference.

In our implementation of the logic-based fuzzy system (see Sect. III-C) the input of the defuzzification process is not a fuzzy set, but a set P of values maximising a theory (the vector P in Algorithm 1.) The output value pos is given by the mean of the values of P.

IV. Simulation and Results

All the results discussed in this section were obtained running our systems on the set of historical data for the markets listed in Table II. Twenty years of historical data (including opening and closing prices) for these markets has been used, except for the Euro/US Dollar market, which has entered the financial markets later. Below we give a concise description of the procedure by means of which the data was obtained.

We first executed each strategy on each market on the whole data set available for that market, in order to obtain the normalised position to assume at each instant for a given market–strategy pair. The next step was then to compute actual financial quantities on an annual basis for a five–years period, January 1, 2003 to January 1, 2008. To increase the number of data generated we further decided to shift forward this annual window at three-month steps. In other words, given a market and a strategy, we fetched data about the position to assume from the database, afterwards using it to calculate needed metrics.

This choice was based upon the fact that when actual trading occurs, we would like to use as much data available as possible. This means referring to information preceding the execution's starting date, if at disposal. This in turn provides a justification to our selection of a common period of execution for all markets (recall for example that data for the Euro/Dollar market did not exist before 1999).

The logic-based fuzzy system was implemented in Java; it is interfaced with **R** (`http://www.r-project.org/`) for technical analysis and plotting. The system has been run with the theories Θ_{MACD} and Θ_{RSI} in Łukasiewicz, Gödel, and Boolean Logic modes. The test with Boolean logic aims to illustrate the limit case of fuzzy indicators used in a Boolean fashion; the results essentially coincide, as expected, with trading based on the non-fuzzy indicators as defined in Sect. II.

TABLE II
THE LIST OF MARKETS (OTC STANDS FOR "OVER THE COUNTER").

Market	Exchange	Quoted In	Ticker
Aluminium	LME	$/t	AH
British Pound/US Dollar	otc	$/£	BP
Brent Crude Oil	IPE	$/bbl.	CO
Euro/US Dollar	otc	$/€	EC
Gold 100 oz.	COMEX	$/oz.	GC
Heating Oil	NYMEX	$/gal.	HO
Live Cattle	CME	¢/lb.	LC
Cocoa	LCE	£/t	QC
S&P 500	CME	pts.	SP
30Y US T-Bond	CBOT	pts.	US

The Mandani-type fuzzy control system was implemented in MATLAB, using the Fuzzy Logic Toolbox along with some auxiliary methods implemented in Java.

Both systems connect up with a MySql DBMS in order to fetch markets' data, and store results.

A. Control surfaces

We present here the control surfaces generated by the implemented systems. Figure 7 shows the control surface of the control system that uses the RSI indicator as is. Note that our implementation slightly modifies the original meaning of the RSI indicator. Namely, while a system based on the latter usually only allows to go long or to go short, our system also allows to stay out of the market at any time (as one can deduce by looking at the flat at level 0.5 in the picture.)

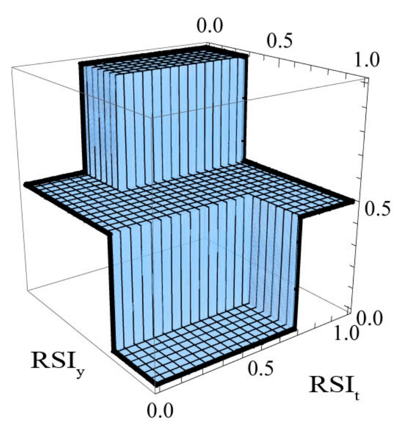

Fig. 7. Control surface for the RSI control system, using Boolean logic's operators.

Next consider Figures 8 and 9. The former shows the control surface of the logic–based system, when configured to use Θ_{RSI} as theory and Gödel Logic as logic, whilst the latter shows the control surface of the Mamdani-type fuzzy control system, as generated by MATLAB's Fuzzy Logic Toolbox. The similarity between the two is evident; for a theoretical explanation of the fact that the logic-based system is indeed more general than the Mamdani-type system, and thus is capable of producing the same control surface computed by the latter, please refer to Part I of the paper.

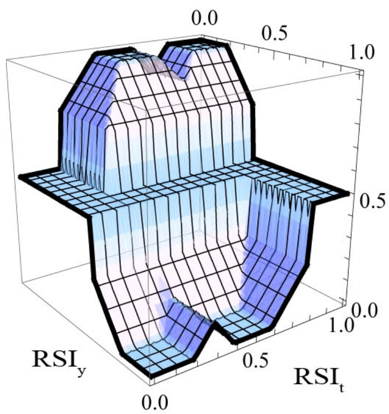

Fig. 8. Control surface for the RSI control system, using Gödel logic's operators.

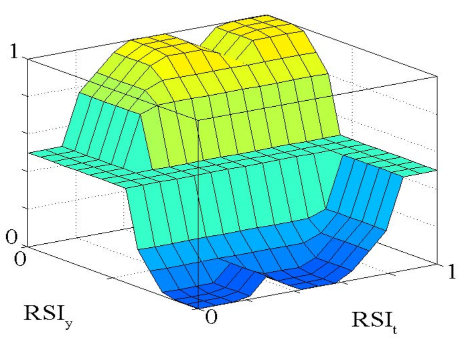

Fig. 9. Control surface for the RSI control system, using Mamdani's operators.

B. Comparison of the Results

Table III provides information about the average percentage returns and volatility for strategies based on the MACD, while the corresponding RSI data are shown in Table IV. The same data is presented visually in Figure 10. It can be observed that, on average, the performance of the logic-based system (both with Łukasiewicz and Gödel logic) is comparable to that of the Mamdani-type system. The same applies to the performance of the systems based on Boolean logic, which, as mentioned, essentially implement standard RSI- or MACD-based trading strategies. Note in addition how, when applying the RSI strategy on some specific markets (CO, GC, HO, SP), the Mamdani-type control system actually performs quite poorly in comparison to all other systems. We do not set forth an explanation for this behaviour in this paper: the data do not question our conclusion that the logic-based systems perform at least as well as the Mamdani-type system in all cases, and this is enough for our purposes here. More experimental research is needed to back up further considerations.

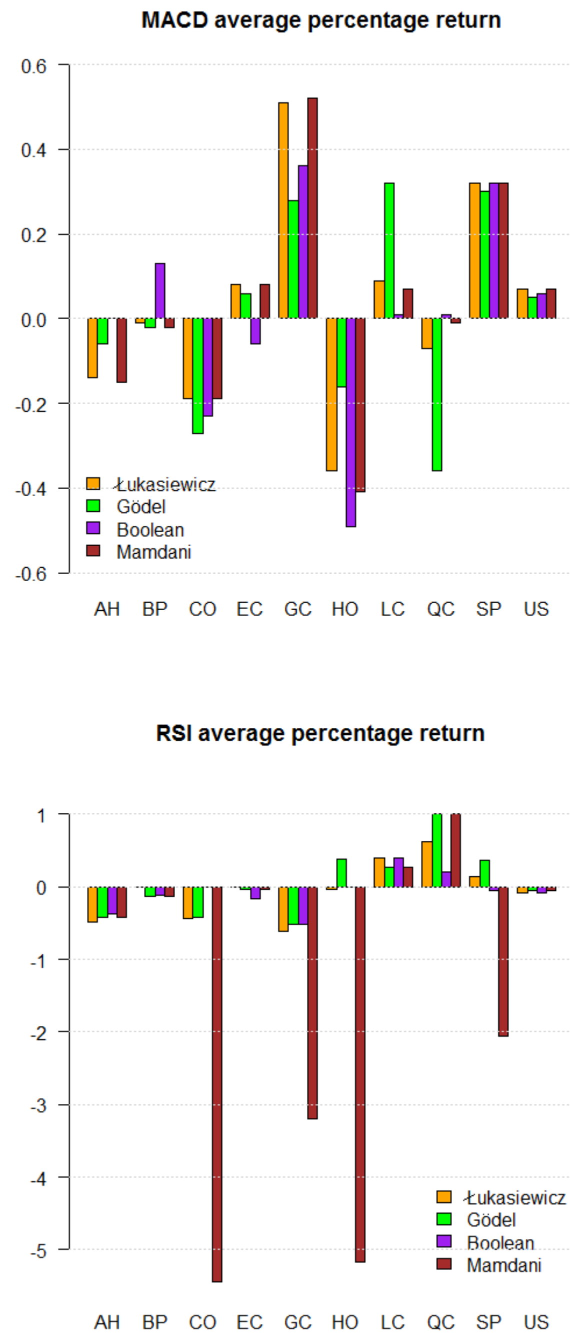

Fig. 10. The average percentage return for the MACD (above) and RSI (below) based strategies.

Finally, for a more immediate comparison among systems, we also reported in Table V the difference computed on average returns for each strategy against the Mamdani-type system. As for volatility, see Table VI.

TABLE III
AVERAGE RETURN AND VOLATILITY FOR THE MACD BASED STRATEGIES.

Mkt	Łukasiewicz		Gödel		Boolean		Mamdani	
	mean	vol.	mean	vol.	mean	vol.	mean	vol.
AH	−0.14	0.47	−0.06	0.24	0.00	0.30	−0.15	0.51
BP	−0.01	0.21	−0.02	0.20	0.13	0.19	−0.02	0.21
CO	−0.19	0.72	−0.27	0.83	−0.23	0.51	−0.19	0.70
EC	0.08	0.20	0.06	0.19	−0.06	0.11	0.08	0.21
GC	0.51	0.53	0.28	0.51	0.36	0.27	0.52	0.53
HO	−0.36	0.67	−0.16	0.63	−0.49	0.68	−0.41	0.69
LC	0.09	0.38	0.32	0.38	0.01	0.22	0.07	0.40
QC	−0.07	0.35	−0.36	0.41	0.01	0.53	−0.01	0.36
SP	0.32	0.21	0.30	0.13	0.32	0.24	0.32	0.21
US	0.07	0.21	0.05	0.21	0.06	0.12	0.07	0.20

TABLE IV
AVERAGE RETURN AND VOLATILITY FOR THE RSI BASED STRATEGIES.

Mkt	Łukasiewicz		Gödel		Boolean		Mamdani	
	mean	vol.	mean	vol.	mean	vol.	mean	vol.
AH	−0.49	0.77	−0.42	0.88	−0.37	0.45	−0.42	0.88
BP	0.00	0.23	−0.13	0.24	−0.12	0.21	−0.13	0.24
CO	−0.44	0.88	−0.42	0.78	0.00	0.00	−5.44	1.78
EC	−0.01	0.29	−0.03	0.49	−0.16	0.19	−0.03	0.49
GC	−0.61	0.37	−0.52	0.51	−0.52	0.58	−3.20	0.74
HO	−0.03	0.57	0.38	0.74	0.00	0.00	−5.17	1.42
LC	0.40	0.59	0.27	1.10	0.39	0.46	0.27	1.10
QC	0.62	0.74	1.01	1.01	0.21	0.40	1.01	1.01
SP	0.14	0.29	0.37	0.34	−0.05	0.16	−2.05	0.47
US	−0.09	0.16	−0.06	0.18	−0.09	0.18	−0.06	0.18

TABLE V
AVERAGE RETURNS DIFFERENCES FOR THE RSI AND MACD BASED STRATEGIES (HIGHER IS BETTER).

Mkt	Łukas. vs. Mamdani		Gödel vs. Mamdani		Bool. vs. Mamdani	
	MACD	RSI	MACD	RSI	MACD	RSI
AH	0.01	−0.07	0.09	0.00	0.15	0.05
BP	0.01	0.13	0.00	0.00	0.15	0.01
CO	0.00	5.00	−0.08	5.02	−0.04	5.44
EC	0.00	0.02	−0.02	0.00	−0.14	−0.13
GC	−0.01	2.59	−0.24	2.68	−0.16	2.68
HO	0.05	5.14	0.25	5.55	−0.08	5.17
LC	0.02	0.13	0.25	0.00	−0.06	0.12
QC	−0.06	−0.39	−0.35	0.00	0.02	−0.80
SP	0.00	2.19	−0.02	2.42	0.00	2.00
US	0.00	−0.03	−0.02	0.00	−0.01	−0.03

V. CONCLUSIONS

We recall that our main objective in this two-part paper was to analyse Mamdani-type fuzzy control systems in logical terms, with special emphasis on the fuzzy inference process. In Part I [1], we provided theoretical arguments to back up our analysis. Further, inspection of the experimental results presented in this second part clearly shows that there are no significant differences in performance between the standard, Mamdani-type implementation of the fuzzy system at hand, and the implementation based on our theoretical analysis in Part I. This lends additional, experimental support to the claim that the main objective set forth in the first part of this paper has been achieved.

TABLE VI
RETURNS' VOLATILITY DIFFERENCES FOR THE RSI AND MACD BASED STRATEGIES (LOWER IS BETTER).

Mkt	Łukas. vs. Mamdani		Gödel vs. Mamdani		Bool. vs. Mamdani	
	MACD	RSI	MACD	RSI	MACD	RSI
AH	−0.04	−0.11	−0.27	0.00	−0.21	−0.43
BP	0.00	−0.01	−0.01	0.00	−0.02	−0.03
CO	0.02	−0.90	0.13	−1.00	−0.19	−1.78
EC	−0.01	−0.20	−0.02	0.00	−0.10	−0.30
GC	0.00	−0.37	−0.02	−0.23	−0.26	−0.16
HO	−0.02	−0.85	−0.06	−0.68	−0.01	−1.42
LC	−0.02	−0.51	−0.02	0.00	−0.18	−0.64
QC	−0.01	−0.27	0.05	0.00	0.17	−0.61
SP	0.00	−0.18	−0.08	−0.13	0.03	−0.31
US	0.01	−0.02	0.01	0.00	−0.08	0.00

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