On Valuations in Gödel and Nilpotent Minimum Logics

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Some decades ago, V. Klee and G.-C. Rota [2,3] introduced a lattice-theoretic analogue of the Euler characteristic, the celebrated topological invariant of polyhedra. In [1], using the Klee-Rota definition, we introduce the Euler characteristic of a formula in Gödel logic, the extension of intuitionistic logic via the prelinearity axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$. We then prove that the Euler characteristic of a formula φ over n propositional variables coincides with the number of Boolean assignments to these n variables that satisfy φ. Building on this, we generalise this notion to other invariants of φ that provide additional information about the satisfiability of φ in Gödel logic. Specifically, the Euler characteristic does not determine non-classical tautologies: the maximum value of the characteristic of $\varphi(X_1, \ldots, X_n)$ is 2^n, and this can be attained even when φ is not a tautology in Gödel logic. By contrast, we prove that these new invariants do.

In this talk, we present the aforementioned results and compare what has been obtained for Gödel logic with analogous results for a different many-valued logic, namely, the logic of *Nilpotent Minimum*. This logic can also be described as the extension of Nelson logic by the prelinearity axiom. The latter results are joint work with D. Valota.

References

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