Combinatorial descriptions of products in the category of forests and open order-preserving maps

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In [1], the authors introduce a technique to compute finite coproducts of finite Gödel algebras, i.e. Heyting algebras satisfying the prelinearity axiom \((\alpha \rightarrow \beta) \lor (\beta \rightarrow \alpha)\). To do so, they investigate the product in the category opposite to finite Gödel algebras: the category of forests and open order-preserving maps. (A forest is a partially ordered set \(F\) such that, for every \(x \in F\), the downset of \(x\) is a chain, when endowed with the order inherited from \(F\)). To achieve their result, the authors make use of ordered partitions of finite sets and of a specific operation – called merged-shuffle – on ordered partitions.

In this talk, besides recalling the aforementioned construction of the product, we show that, from an enumerative point of view, such a product can be simply described in terms of Delannoy coefficients, of bipartite graphs, and of product of matrices.

Bibliography