The independent subsets of powers of paths and cycles

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For a graph $\mathbf{G}$ we denote by $V(\mathbf{G})$ the set of its vertices, and by $E(\mathbf{G})$ the set of its edges. For $n, h \geq 0$,

(i) the *h-power of a path*, denoted by $\mathbf{P}_n^{(h)}$ is a graph with n vertices $v_1, v_2, \ldots, v_n$ such that, for $1 \leq i, j \leq n$, $(v_i, v_j) \in E(\mathbf{P}_n^{(h)})$ if and only if $|j - i| \leq h$;

(ii) the *h-power of a cycle*, denoted by $\mathbf{Q}_n^{(h)}$ is a graph with n vertices $v_1, v_2, \ldots, v_n$ such that, for $1 \leq i, j \leq n$, $(v_i, v_j) \in E(\mathbf{Q}_n^{(h)})$ if and only if $|j - i| \leq h$ or $|j - i| \geq n - h$.

Thus, for instance, $\mathbf{P}_n^{(0)}$ and $\mathbf{Q}_n^{(0)}$ are the graphs made of n isolated nodes, $\mathbf{P}_n^{(1)}$ is the path with n vertices, and $\mathbf{Q}_n^{(1)}$ is the cycle with n vertices.

The following definition introduces the key notion in this work.

Definition *An* independent subset S *of a graph* $\mathbf{G}$ *is a subset of* $V(\mathbf{G})$ *not containing adjacent vertices.*

In the first part of our work we deal with the *independent subsets* of $\mathbf{P}_n^{(h)}$. We evaluate $p_n^{(h)}$, i.e. the number of independent subsets of $\mathbf{P}_n^{(h)}$, and $H_n^{(h)}$, i.e. the number of edges of the Hasse diagram of the poset of independent subsets of $\mathbf{P}_n^{(h)}$ ordered by inclusion.

Our main result is that, for $n, h \geq 0$, the sequence $H_n^{(h)}$ is the convolution of the sequence $p_n^{(h)}$ with itself.

In the second part, we derive similar results for the number of edges of the Hasse diagram of the poset of independent subsets of $\mathbf{Q}_n^{(h)}$.

If time allows, we discuss some bijections connecting our independent subsets with certain compositions of an integer.