The Euler Characteristic of a formula in many-valued logic

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Some decades ago, V. Klee and G.-C. Rota introduced a lattice-theoretic analogue of the Euler characteristic, the celebrated topological invariant of polyhedra. Using the Klee-Rota definition, we introduce the Euler characteristic of a formula in Gödel logic (an extension of intuitionistic logic, or, equivalently, a many-valued logic). We then prove that the Euler characteristic of a formula φ coincides with the number of Boolean assignments that satisfy φ. Building on this, we generalise this notion to other invariants of φ that provide additional information about the satisfiability of φ in Gödel logic. Specifically, while the Euler characteristic does not determine non-classical tautologies, these new invariants do. Finally, if time allows, we sketch a similar analysis for other many-valued logics.

References

[1] P. Codara, O. M. D'Antona, V. Marra, *The Euler Characteristic of a Formula in Gödel Logic*, IEEE International Symposium on Multiple-Valued Logic (ISMVL) 2010, in press.

[2] V. Klee, *The Euler characteristic in combinatorial geometry*, Amer. Math. Monthly **70** (1963), 119–127

[3] E. Munarini, *On the Euler characteristic of Finite Distributive Lattices*, In: From Combinatorics to Philosophy: The Legacy of G.-C. Rota, Springer, New York (2009), 145–166.

[4] G.-C. Rota, *On the combinatorics of the Euler characteristic*, Studies in Pure Mathematics, Academic Press, London (1971), 221–233.